46. Proposed by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

"There was an old woman tossed up in a basket
Ninety times as high as the moon." *Mother Goose.*

Neglecting the resistance of the air, how long did it take the old lady to go up ?

I. Solution by E. L. SHERWOOD, A. M., Superintendent of City Schools, West Point, Mississippi.

The equation of motion is

$$1.\quad \frac{d^2s}{dt^2}=-\frac{gr^2}{s^2}.\qquad 2.\quad \left(\frac{ds}{dt}\right)^2=\frac{2gr^2}{s}+c$$

where ds/dt or $v=0$, when $S=90\times 60.3R$ or $5427R$.

Whence $C=-\dfrac{2gr^2}{5427R}$.

$$3.\quad \left(\frac{ds}{dt}\right)^2=\frac{2gr^2}{s}-\frac{2gr^2}{5427r}\text{ or }2gr^2\left(\frac{1}{s}-\frac{1}{a}\right).$$

$$4.\quad dt=\sqrt{\frac{a}{2gr^2}}\cdot\frac{sds}{\sqrt{as-s^2}}\text{ solving }dt\text{ in (3).}$$

$$5.\quad t=\sqrt{\frac{a}{2gr^2}}\int_a^R\frac{sds}{\sqrt{as-s^2}}\text{ for }t=0\text{ when }s=a.$$

$$6.\quad t=\sqrt{\frac{a}{2gr^2}}\left[\left(\sqrt{as-s^2}-\tfrac{1}{2}a\text{vers}^{-1}\frac{2s}{a}+C\right)\right]_a^R.$$

$$7.\quad t=\sqrt{\frac{a}{2gr^2}}\left(\sqrt{aR-R^2}-\tfrac{1}{2}a\text{vers}^{-1}\frac{2R}{a}+\frac{\pi a}{2}\right)\text{ where }a=5427R.$$

8. $t=11.35+$ years, by substituting values and reducing.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Let t=time, $R=3963$ miles$=20924640$ feet=radius of the earth, $g=32.2$ feet=gravity, $a=90(60R)=5400R$=distance the old woman was tosssd.

$$\therefore t=\sqrt{\frac{a}{2gR^2}}\left(\sqrt{aR-R^2}-\tfrac{1}{2}a\text{vers}^{-1}\frac{2R}{a}+\frac{\pi a}{2}\right).$$

$$t=30\sqrt{\frac{3}{gR}}\{R\sqrt{5399}-2700R\text{vers}^{-1}\tfrac{1}{2700}+2700\pi R\}.$$

$t=355287708.316$ seconds$=11$ years, 3 months, 7 days, 3 hours, 1 minute, 48.316 seconds.

Also solved by *J. C. CORBIN.*

47. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

What is the focus of the convex surface of a plano-convex lens, index μ, which will converge parallel monochromatic rays to a given focus, the rays entering the lens on the plane side?

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Let f=the given focal length. F=the focal length required,

u=distance of origin of ray from lense,

r, s, the radii of the first and second surfaces of the lense respectively,

t=the thickness, and regard all distances as measured from the posterior surface.

Then we have for for a double convex lense,

$$\frac{1}{\frac{1}{f}+\frac{\mu-1}{s}}-\frac{1}{\frac{1}{u}-\frac{\mu-1}{r}}=\frac{t}{\mu}.$$

(See Parkinson's Optics, Art. 100, Cor. I, page 91).

Let $u=r=\infty$.

$$\therefore \frac{1}{\frac{1}{f}+\frac{\mu-1}{s}}=\frac{t}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

This is the plano-convex lense with light incident upon plane surface.

Write F for f, and let $s=u=\infty$.

$$\therefore \frac{1}{\frac{1}{F}}+\frac{1}{\frac{\mu-1}{r}}=\frac{t}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

This is the plano-convex lense with light incident upon the convex surface. Since we are using the same lense, $r=s$.

$$\therefore\ r=z=\frac{(\mu-1)ft}{\mu f-t},\ \text{from (1)}.$$

This value of r in (2) gives, $F=-\dfrac{t^2(\mu-1)}{(\mu f-t)^2}$.

$\therefore$ F is found independent of the radius of convexity.